1. For a group of 7 people, find the probability that all 4 seasons (winter, spring, summer, fall) occur at least once each among their birthdays, assuming that all seasons are equally likely.

Ans:- Given that seasons are equally likely and all 4 seasons (winter, spring, summer, fall) occur at least once each among their birthdays

Total outcomes: 12\*4=48

Using Inclusion and Exclusion, 4C1\*38 - 4C2\*28 + 4C3\*18  [i.e. Exclude 1 season - Exclude 2 season + exclude 3 season, also note that we can't exclude all the 4 seasons]

Probability that one or more season has no student having their birthday:

(4C1\*38 - 4C2\*28 + 4C3\*18  ) / 48 = 0.377

Required probability that all 4 seasons (winter, spring, summer, fall) occur at least once each among their birthdays:

1-0.377=0.623

2.Alice attends a small college in which each class meets only once a week. She is deciding between 30 non-overlapping classes. There are 6 classes to choose from for each day of the week, Monday through Friday. Trusting in the benevolence of randomness, Alice decides to register for 7 randomly selected classes out of the 30, with all choices equally likely. What is the probability that she will have classes every day, Monday through Friday?

Ans:- Direct Method: There are two general ways that Alice can have class every day: either she has 2 days with 2 classes and 3 days with 1 class, or she has 1 day with 3 classes, and has 1 class on each of the other 4 days. The number of possibilities for the former is 5 2 6 2 2 63 (choose the 2 days when she has 2 classes, and then select 2 classes on those days and 1 class for the other days). The number of possibilities for the latter is 5 1 6 3 64. So the probability is 5 2 6 2 2 63 + 5 1 6 3 64 /30 7 = 114 /377 ~ 0.302.

Inclusion-Exclusion Method: we will use inclusion-exclusion to find the probability of the complement, which is the event that she has at least one day with no classes. Let Bi = Ac i.

then P(B1 U B2 U··· [ B5) = ∑ P(Bi)-∑P(Bi∩Bj)+∑P(Bi∩Bj∩Bk)

(terms with the intersection of 4 or more Bi’s are not needed since Alice must have classes on at least 2 days). We have

P(B1) = 24 7 30 7 , P(B1 ∩B2) = 18 7 30 7 , P(B1 ∩B2 ∩B3) = 12 7 /30 7

and similarly for the other intersections. So

P(B1 U ··· [ B5)=5 24 7 30 7 -5 2 ◆18 7 30 7 + 5 3 ◆12 7 30 7 = 263 /377.

Therefore,

P(A1 ∩ A2 ∩ A3 ∩A4 ∩ A5) = 114 377 ~ 0.302.